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George W. Ladd
Iowa State University

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Nonoptimality Of A Simple And Common Method For Determining Patronage Refunds

Abstract

It may help to avoid some confusion later by noting now that equation (1) is not a definition of patronage refund per unit of product j . Equation (1) is a method of determining patronage refunds; other methods also exist. Two other methods of determining patronage refunds will be presented. These are not to be interpreted as recommended methods...

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NONOPTIMALITY OF A SIMPLE AND COMMON
METHOD FOR DETERMINING PATRONAGE REFUNDS

by

George W. Ladd

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One method used by cooperatives for determining patronage refunds to members is based on (1).

$$(1) \ r_j = p_j - v_j - f_v \geq 0$$

where

r_j = patronage refund paid to members per unit of j-th product.

p_j = price charged for j-th product.

v_j = average variable cost for product j.

f_j = average fixed cost allocated to product j.

The purpose of this note is to show that this simple and intuitively reasonable method is inconsistent with the attainment of various reasonable cooperative objectives. Two different cooperative objectives are considered: (a) maximization of total patronage refunds, and (b) maximization of a utility function containing total refunds and quantities of various "prestige" products sold. The inconsistency of (1) with objective (a) is proved in two independent ways: (A) by using theory of the multi-product firm, and (B) by considering the effect of adding constraints to a maximization problem. The inconsistency of (1) with objective (b) is proved by method (B).

The analysis is static and the cooperative's only business is assumed to be selling supplies to members; it does not buy from members.

DEFINITION OF PATRONAGE REFUND

It may help to avoid some confusion later by noting now that equation (1) is not a definition of patronage refund per unit of product j. Equation (1) is a method of determining patronage refunds; other methods also exist. Two other methods of determining patronage refunds will be presented. These are not to be interpreted as

recommended methods. They are presented simply to emphasize the point that (1) is a method and cannot be a definition because other methods are available that satisfy the same definition of "patronage refund per unit of project j " that (1) satisfies. It can also be shown that each of these methods is inconsistent with the attainment of reasonable objectives.

The first method determines patronage refunds for all products as equal proportions of product prices. Let q_i be quantity sold of product i , and let V and F be total variable cost and total fixed cost and let π be total patronage refund. Compute $\pi = \sum_i p_i q_i - V - F$. Define $\rho = \pi / \sum_i p_i q_i$ and determine the patronage refund per unit of product j as $r_j = \rho p_j$. Then $\sum_i r_i q_i = \rho \sum_i p_i q_i = \pi$.

The second method results in the same patronage refund per unit of product for all products. Compute ρ_1 from $\rho_1 = \pi / \sum_j q_j$ and set $r_j = \rho_1$ for all j . Then $\sum_j r_j q_j = \rho_1 \sum_j q_j = \pi$.

EFFECT OF ADDING CONSTRAINTS TO A MAXIMIZATION PROBLEM

In this section it is shown that the addition of a constraint to a maximization problem cannot increase the maximum attainable value of the objective function, and can decrease its maximum value.

This seems intuitively reasonable. But I present a proof because many things that I have thought to be intuitively obvious have turned out to be demonstrably logically false.

Suppose it is desired to maximize $f(X) = f(x_1, x_2, \dots, x_n)$ subject to m independent constraints $c_i - b_i(X) = 0$ for $i = 1, 2, \dots, m$. Each c_i is a scalar constant. Each $b_i(X) = b_i(x_1, x_2, \dots, x_n)$ is a function of the x_j . The "opportunity set" or "feasible set" is the

set of points X satisfying all the constraints; denote this set by K_1 .

$$K_1 = \{X \mid c_i - b_i(X) = 0 \text{ for } i = 1, 2, \dots, m\}$$

Assume X^* is feasible (i.e., $X^* \in K_1$) and is a global maximum. Then

$$(2) f(X^*) \geq f(X^{**}) \text{ for all } X^{**} \in K_1, X^{**} \neq X^*$$

Now suppose another constraint is added to the problem:

$c_{m+1} - b_{m+1}(X) = 0$. The opportunity set or feasible set for this new problem is K_2 .

$$K_2 = \{X \mid c_i - b_i(X) = 0 \text{ for } i = 1, 2, \dots, m+1\}$$

This set can also be written

$$K_2 = \{X \mid c_i - b_i(X) = 0 \text{ for } i = 1, 2, \dots, m \text{ and } c_{m+1} - b_{m+1}(X) = 0\}$$

Consider also the set K_3

$$K_3 = \{X \mid c_i - b_i(X) = 0 \text{ for } i = 1, 2, \dots, m \text{ and } c_{m+1} - b_{m+1}(X) \neq 0\}$$

It can be seen that K_1 is the union of K_2 and K_3 : Every point in K_2 or K_3 is in K_1 .

$$(3) K_1 = K_2 \cup K_3$$

Suppose $f(Z)$ is the maximum value of the objective function for this second problem, then

$$Z \in K_2, f(Z) \geq f(Z^*) \text{ for all } Z^* \in K_2, Z \neq Z^*$$

Is it possible for $f(Z)$ to be greater than or equal to $f(X^*)$? The answer is clearly "no." By (3), K_2 is a subset of K_1 . Therefore $Z \in K_1$. Because (2) holds for all $X^{**} \in K_1$, it also holds for the particular X^{**} obtained by setting $X^{**} = Z$. Therefore $f(X^*) \geq f(Z)$.

Suppose that we have, instead of (2),

$$(2a) f(X^*) = f(Y) > f(X') \text{ for all } X' \in K_1, X^* \neq Y \neq X', Y \in K_3$$

Two different feasible points maximize $f(X)$: the points X^* and Y . Now, Y is not in the feasible set for the second problem. And K_2 is a subset of K_1 . Therefore,

$$f(X^*) > f(X') \text{ for all } X' \in K_2, X^* \neq X'$$

and adding the constraint has reduced the maximum attainable value of the objective function.

Adding a constraint cannot increase the maximum value of the objective function. The only effect of adding a restriction is to reduce the size of the feasible set, i.e., to eliminate some points from the feasible set. In this latter case, adding the $m+1$ -st constraint eliminated all of the points in K_3 from the feasible set.

This purely mathematical result has an important implication for cooperatives. It says something about ways in which patronage refunds on individual products should not be determined if the cooperative is interested in the total patronage refunds paid to members.

MAXIMIZATION OF PATRONAGE REFUNDS

First Proof of Nonoptimality of (1)

Consider a cooperative whose sole business is selling supplies to members: it buys nothing from members. Let p_i and q_i be price charged for and quantity sold of product i ; let the cooperative's total variable cost function be $V(q_1, q_2, \dots, q_n)$, and let the cooperative's total fixed cost be F . Total patronage refunds, then, are

$$(4) \pi = \sum_{i=1}^n p_i q_i - V(q_1, q_2, \dots, q_n) - F$$

From the argument of the preceding section, it immediately follows that using (1) to determine refunds limits the attainable value of π . Maximizing π while using (1) is equivalent to solving the problem:

Maximize π subject to $r_i = p_i - v - f_i \geq 0$ for $i = 1, 2, \dots, m$
 and subject to $\sum_i f_i q_i = F$

The use of (1) amounts to adding a number of restrictions to the problem, and therefore can be expected to reduce the maximum attainable value of π .

This mathematical argument, though valid, provides no economic insight. The following argument provides some economic insight into the problem.

Second Proof of Nonoptimality of (1)

The first-order conditions for maximizing (4) with respect to prices are the n relations

$$(5) \quad \partial\pi/\partial p_j = 0 = q_j + \sum_{i=1}^n p_i \partial q_i / \partial p_j - \sum_{i=1}^n (\partial V / \partial q_i) (\partial q_i / \partial p_j) \\ j = 1, 2, \dots, n$$

The margin of price over marginal cost for the j -th product is obtained by solving (5) for $p_j - \partial V / \partial q_j$. It is

$$(6) \quad p_j - \partial V / \partial q_j = -[q_j + \sum_{i \neq j} (p_i - \partial V / \partial q_i) (\partial q_i / \partial p_j)] / (\partial q_j / \partial p_j)$$

This is exactly the same relation that Holdren obtained for margin in his study of multiproduct firms. Relation (6) establishes that "...

if the dominant relationship is one of complementarity ($\sum_{i \neq j} (p_i - \partial V / \partial q_i) \partial q_i / \partial p_j < 0$) the profit margin on the j -th commodity may be zero, greater than zero or less than zero."¹ If many of the products that the cooperative sells are highly complementary with product j , ($\partial q_i / \partial p_j < 0$), then (6) may well be negative.

¹ Holdren, Bob R. The Structure of a Retail Market and the Market Behavior of Retail Units. Ames, Iowa. Iowa State University Press, 1958, pp. 128-129.

To see the implications of (6) for using (1) to determine refunds, consider first a simple case in which $V = \sum_{i=1} v_i q_i$ and all v_i are constant, i.e., $\partial v_i / \partial q_j = 0$ for all i and j . Then $\partial V / \partial q_i = v_i$ and (6) becomes

$$(7) \quad p_j - v_j = - \left[\sum_{i \neq j} (p_i - v_i) (\partial q_i / \partial p_j) + q_j \right] / (\partial q_j / \partial p_j)$$

Assume other products are highly complementary with product j . Then

$$(8) \quad p_j - v_j < 0$$

and $r_j = p_j - v_j - f_j < 0$ (assuming $f_j \geq 0$), violates (1). If the cooperative requires $p_j - v_j - f_j \geq 0$ when (8) holds, it is clear that (7) is not being satisfied. And if (7) is not satisfied, the cooperative is not maximizing total patronage refunds because the first-order conditions (6) are not being satisfied.

If $V = \sum_{i=1} v_i q_i$ and $\partial V / \partial q_j = v_j + \sum_h q_h \partial v_h / \partial q_j$, (6) can be written

$$(9) \quad p_j - v_j = - \left[q_j + \sum_{i \neq j} (p_i - \partial V / \partial q_i) (\partial q_i / \partial p_j) \right] / (\partial q_j / \partial p_j) \\ + \sum_h q_h \partial v_h / \partial q_j$$

Now if q_j is complementary with many other products in production ($\partial v_h / \partial q_j < 0$) and if demands are highly complementary, again $p_j - v_j$ may well be negative.

Even if $p_j - v_j > 0$, using (1) imposes a restriction: namely that $p_j - v_j \geq f_j$.

UTILITY MAXIMIZATION

Maximization of patronage refunds is not the only plausible objective for a cooperative. A cooperative may also desire large volumes of sales of certain products. This may occur because the cooperative acquires prestige from sales of these products, because these products are expected to be highly profitable in the future even though they may not be profitable now, or for some other reason.

Suppose the cooperative's objective is not to maximize (4), but to maximize (4a)

$$(4a) U = U(\pi; q_1, q_2, \dots, q_m)$$

It is still true that use of (1) for determining the patronage refunds is inconsistent with the cooperative's objectives because using (1) imposes restrictions. Maximizing (4a) and using (1) is equivalent to solving:

$$(10) \text{ Maximize } U\left(\sum_{i=1}^m r_i q_i, q_1, q_2, \dots, q_m\right)$$

subject to $r_i = p_i - v_i - f_i \geq 0$ for all i .

AN IMPLICATION OF THEORY OF THE MULTI-PRODUCT FIRM

Expressions (6) through (9) have some important implications for multi-product cooperatives: implications that run counter to what some people conceive to be sound cooperative operating procedure. Recently the University of Georgia Cooperative Extension Service carried out a study whose objective was ". . . to develop a guide for formulating and evaluating company policies at the board of director level in agricultural cooperatives." (p. 3)² In carrying out the study, they used a 12-page questionnaire in a survey of cooperative managers, boards of directors and members. I quote the complete discussion of the topic of Financial Support of New Services.³

²Policy Formulation for Agricultural Cooperatives at the Board of Director Level, Athens, Georgia, 1973.

³Op. cit., p. 58. The reference to Gardner refers to Gardner, Kelsey B. News for Farmer Cooperatives, "Are New Operating Concepts Compatible Bedfellows with Basic Co-op Principles?" FCS, USDA, Nov. 1971.

"Financial Support of New Services

Management cannot predict with certainty which new services will actually be self-supporting, but it is obliged to try, since it is obviously unfair to members who do not want or need a new service to find themselves subsidizing it from their other cooperative enterprises. The question formulated for the Survey was as follows:

In order to avoid the inequity of one service of a cooperative subsidizing some other service, diligent effort should be made to insure that any new service offered will be able to carry its own costs.

There was 91 percent approval of this policy.

When a cooperative finds itself with a new enterprise that cannot carry its own costs, naturally the deficit must be made up from the more profitable enterprises. The question of how long a losing enterprise should be subsidized needs a policy statement. This one was formulated for the Survey:

In the event that one department or division operates at a loss, the deficit may be borrowed from the savings of other departments, but it should be repaid to the lending department(s) at the earliest date.

There was 79 percent approval and only 9 percent disapproval of this policy.

Certainly no activity of the cooperative should be subsidized indefinitely from other enterprises or lines of supply. Gardner refers to this practice as "netting" and has this to say about it:

I believe that "netting" of losses is not wholly consistent with the operation-at-cost principle, as applied to the individual member's transactions. However, I believe also that the door should not be completely closed to an

association that desires to use "netting" procedure where circumstances warrant, for example, absorbing some of the usually inevitable developmental deficits of a new service before it has achieved its full operating potential."

Consider the statement, "Certainly no activity of the cooperative should be subsidized indefinitely from other enterprises or lines of supply" in conjunction with expressions (7) and (8). If the dominant relationship is one of complementarity with project j , then $p_j - v_j < 0$. Does the quotation mean that product j (assume it to be an established product) should be dropped? If it means this, then it means that the cooperative is being advised to cut down on its total patronage refunds. How come it means this? If $p_j - v_j < 0$ and $x_j > 0$, the cooperative is being advised: "Set x_j at zero." Setting $x_j = 0$ is, in effect, maximizing (4) subject to the restriction $x_j = 0$. And, as we saw earlier, adding restrictions cannot increase the maximum value of the objective function; and can be expected to reduce its value.

In discussing a product for which the right-hand side of (6) -- or (7) or (9) -- is negative, we are discussing a "loss-leader" in the terminology of retailing. Many other products are highly complementary with this product. Because of the many highly negative values of $\partial q_i / \partial p_j$, a low price on this product substantially increases sales volume of a number of other products. And the addition to profits resulting from the added sales of these other products more than compensates for any loss on the loss-leader. Considered in isolation from all other products, a loss-leader may be a money-loser. But when its complementary relationship to other products is taken into account, it is a money-maker.

If we, as a profession, are telling cooperatives to drop their

loss-leaders, are we telling them, "Let your proprietary competitors have all the loss-leaders. Let them have all these products that successfully attract additional business."

WHAT NEXT?

Assuming that one accepts the argument of this paper that using (1) to determine refunds on individual products limits the total amount of refunds a cooperative can earn (and pay), the question must arise: "What to do next?"

Writers on science (and on scientists) have noted that "no theory cannot drive out bad theory." That is, one cannot induce scientists to stop using a theory simply by convincing them that it is a bad theory. To induce scientists to stop using a bad theory, it is necessary to convince them that it is a bad theory and also to present them a superior alternative. I daresay that something similar is true of businessmen: "No decision rule cannot drive out a bad decision rule." To get businessmen to stop using a bad decision rule, it is necessary to convince them that it is a bad rule and also to present them a superior alternative.

The answer to the question "What next?" seems to involve two things. (A) Determine the proper place for loss-leaders in cooperative enterprises. (B) Develop an alternative to (1) that is superior to (1), relatively easy to use, and consistent with the decision made on loss-leaders in (A) and with cooperatives' objectives.